Sequences and Series Pretest

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Name _____   Date _____
Algebra 2 Unit 11: Sequence and Series

Sequences and Series Pretest

1. Given each of the following sequences defined by formulas, determine and label the first four terms. A variety of different notations is used below for practice purposes.

(a) $f(n) = 7n + 2$ (b) $a_n = n^2 - 5$ (c) $t(n) = \left(\frac{2}{3}\right)^n$ (d) $t_n = \dfrac{1}{n+1}$

(a)
9
16
23
30

(b)
-4
-1
4
11

(c)
2/3
4/9
8/27
16/81

(d)
1/2
1/3
1/4
1/5

2. Sequences below are defined recursively. Determine and label the **next** three terms of the sequence.

(a) $f(1) = 4$ and $f(n) = f(n-1) + 8$

12
20
28

(b) $a(n) = a(n-1) \cdot \frac{1}{2}$ and $a(1) = 24$

12
6
3

(c) $b_n = b_{n-1} + 2n$ with $b_1 = 5$

9
15
23

(d) $f(n) = 2f(n-1) - n^2$ and $f(1) = 4$

4
-1
-18

3. Given the sequence 7, 11, 15, 19, ..., which of the following represents a formula that will generate it? *HINT: Guess and check.*

(1) $a(n) = 4n + 7$ (3) $a(n) = 3n + 7$

(2) $a(n) = 3n + 4$ (4) $a(n) = 4n + 3$ ⟵ circled

1

4. A recursive sequence is defined by $a_{n+1} = 2a_n - a_{n-1}$ with $a_1 = 0$ and $a_2 = 1$. Which of the following represents the value of a_5 ?

(1) 8   (3) 3

(2) -7   (4) 4

5. Which of the following formulas would represent the sequence 10, 20, 40, 80, 160, …

(1) $a_n = 10^n$ (3) $a_n = 5(2)^n$

(2) $a_n = 10(2)^n$ (4) $a_n = 2n + 10$

6. For each of the following sequences, determine an algebraic formula that defines the sequence.

(a) 5, 10, 15, 20, …   (b) 3, 9, 27, 81, …   (c) $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dots$

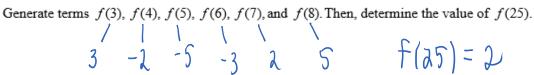$a(n) = 5n$ $a(n) = 3^n$ $a(n) = \dfrac{n}{n+1}$

7. For each of the following sequences, state a recursive definition.

(a) 8, 6, 4, 2, …   (b) 2, 6, 18, 54, …   (c) 2, -2, 2, -2, …

$a_1 = 8$ $a_1 = 2$ $a_1 = 2$

$a_n = a_{n-1} - 2$ $a_n = a_{n-1} \cdot 3$ $a_n = -a_{n-1}$

8. Consider a sequence defined similarly to the Fibonacci, but with a slight twist:

$$f(n) = f(n-1) - f(n-2) \text{ with } f(1) = 2 \text{ and } f(2) = 5$$

Generate terms $f(3)$, $f(4)$, $f(5)$, $f(6)$, $f(7)$, and $f(8)$. Then, determine the value of $f(25)$.

3   -2   -5   -3   2   5 $f(25) = 2$

2

9. Generate the next **three** terms of each arithmetic sequence shown below.

(a) $a_1 = -2$ and $d = 4$

$2, 6, 10$

(b) $f(n) = f(n-1) - 8$ with $f(1) = 10$

$2, -6, -14$

(c) $a_1 = 3$, $a_2 = 1$

$-1, -3, -5$

10. In an arithmetic sequence $t_n = t_{n-1} + 7$. If $t_1 = -5$ determine the values of t_4 and t_{20}. Show the calculations that lead to your answers.

$t_4 = 16$ $t_{20} = 128$

11. If $x + 4$, $2x + 5$, and $4x + 3$ represent the first three terms of an arithmetic sequence, then find the value of x. What is the fourth term?

$x = 3$ $a_4 = 19$

12. If $f(1) = 12$ and $f(n) = f(n-1) - 4$ then which of the following represents the value of $f(40)$?

(1) -148

(2) -140

(3) -144

(4) -172

13. In an arithmetic sequence of numbers $a_1 = -4$ and $a_6 = 46$. Which of the following is the value of a_{12}?

(1) 120

(2) 146

(3) 92

(4) 106

3

14. The first term of an arithmetic sequence whose common difference is 7 and whose 22nd term is given by $a_{22} = 143$ is which of the following?

(1) -25

(3) 7

(2) -4 ⟨circled⟩

(4) 28

15. Generate the next **three** terms of each geometric sequence defined below.

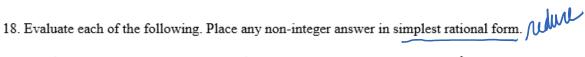(a) $a_1 = -8$ with $r = -1$

$8, -8, 8$

(b) $a_n = a_{n-1} \cdot \frac{3}{2}$ and $a_1 = 16$

$24, 36, 54$

(c) $f(n) = f(n-1) \cdot -2$ and $f(1) = 5$

$-10, 20, -40$

16. Given that $a_1 = 5$ and $a_2 = 15$ are the first two terms of a geometric sequence, determine the values of a_3 and a_{10}. Show the calculations that lead to your answers.

$a_3 = 45$ $a_{10} = 98,415$

17. In a geometric sequence, it is known that $a_1 = -1$ and $a_4 = 64$. The value of a_{10} is

(1) -65,536 ⟨marked⟩

(3) 512

(2) 262,144 ⟨circled⟩

(4) -4096

4

18. Evaluate each of the following. Place any non-integer answer in simplest rational form. *reduce*

(a) $\sum_{i=2}^{5} 4i$

56

(b) $\sum_{k=0}^{3} (k^2 + 1)$

18

(c) $\sum_{j=-2}^{0} (2j + 1)$

-3

(d) $\sum_{i=-1}^{3} 2^i$

$\dfrac{31}{2}$

(e) $\sum_{k=0}^{2} (-1)^{2k+1}$

-3

(f) $\sum_{i=1}^{3} \log(10^i)$

6

(g) $\sum_{n=1}^{4} \dfrac{n}{n+1}$

$\dfrac{163}{60}$

(h) $\dfrac{\sum_{i=2}^{4} (i+1)^2}{\sum_{i=2}^{4} (i^2 + 1)}$

$\dfrac{25}{16}$

(i) $\sum_{k=0}^{3} 256^{\frac{1}{2^k}}$

278

19. Which of the following is the value of $\sum_{k=0}^{4} (4k + 1)$?

(1) 53

(2) 45

(3) 37

(4) 80

20. The sum $\sum_{i=4}^{7} 2^{i-7}$ is equal to

(1) $\dfrac{15}{8}$
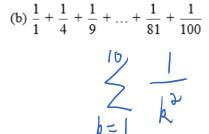
(2) $\dfrac{3}{2}$

(3) $\dfrac{3}{4}$

(4) $\dfrac{7}{8}$

5

21. Write each of the following sums using sigma notation. Use k as your index variable. Note, there are many correct ways to write each sum (and even more incorrect ways).

(a) $-2 + 4 + -8 + \ldots + 64 + -128$

$$\sum_{k=1}^{7} (-2)^k$$

(b) $\dfrac{1}{1} + \dfrac{1}{4} + \dfrac{1}{9} + \ldots + \dfrac{1}{81} + \dfrac{1}{100}$

$$\sum_{k=1}^{10} \frac{1}{k^2}$$

(c) $4 + 9 + 14 + \ldots + 44 + 49$

$$\sum_{k=0}^{9} (4 + 5k)$$

22. Which of the following represents the sum $2 + 5 + 10 + \ldots + 82 + 101$?

(1) $\displaystyle\sum_{j=1}^{6}(4j-3)$

(3) $\displaystyle\sum_{j=1}^{10}(j^2+1)$ ⟲

(2) $\displaystyle\sum_{j=3}^{103}(j-2)$

(4) $\displaystyle\sum_{j=0}^{11}(4^j+1)$

23. A sequence is defined recursively by the formula $b_n = 4b_{n-1} - 2b_{n-2}$ with $b_1 = 1$ and $b_2 = 3$.

What is the value of $\displaystyle\sum_{i=3}^{5} b_i$? Show the work that leads to your answer.

160

24. Which of the following represents the sum of $3 + 10 + \ldots + 87 + 94$ if the arithmetic series has 14 terms?

(1) 1,358

(3) 679 ⟲

(2) 658

(4) 1,276

6

25. The sum of the first 50 natural numbers is
(1) 1,275 *(circled)*
(2) 1,875
(3) 1,250
(4) 950

26. If the first and last terms of an arithmetic series are 5 and 27, respectively, and the series has a sum 192, then the number of terms in the series is
(1) 18
(2) 11
(3) 14
(4) 12 *(circled)*

27. Find the sum of each arithmetic series described or shown below.

(a) The sum of the first 100 even, natural numbers.

10,100

(b) The sum of multiples of five from 10 to 75, inclusive.

595

(b) A series whose first two terms are -12 and -8, respectively, and whose last term is 124.

1,960

(d) A series of 20 terms whose last term is equal to 97 and whose common difference is 5.

990

28. For an arithmetic series that sums to 1,485, it is known that the first term equals 6 and the last term equals 93. *Algebraically* determine the number of terms summed in this series.

30

7

29. Arlington High School recently installed a new black-box theatre for local productions. They only had room for 14 rows of seats, where the number of seats in each row constitutes an arithmetic sequence starting with eight seats and increasing by two seats per row thereafter. How many seats are in the new black-box theatre? Show the calculations that lead to your answer.

$$294$$

30. Simeon starts a retirement account where he will place $50 into the account on the first month and increasing his deposit by $5 per month each month after. If he saves this way for the next 20 years, how much will the account contain in principal?

$$155,400$$

31. The distance an object falls per second while only under the influence of gravity forms an arithmetic sequence with it falling 16 feet in the first second, 48 feet in the second, 80 feet in the third, etcetera. What is the total distance an object will fall in 10 seconds? Show the work that leads to your answer.

$$1600$$

32. A large grandfather clock strikes its bell once at 1:00, twice at 2:00, three times at 3:00, etcetera. What is the total number of times the bell will be struck in a day? Use an arithmetic series to help solve the problem and show how you arrived at your answer.

$$156$$

8

33. Find the sums of geometric series with the following properties:

(a) $a_1 = 6$, $r = 3$ and $n = 8$

19,680

(b) $a_1 = 20$, $r = \frac{1}{2}$, and $n = 6$

39,375

(c) $a_1 = -5$, $r = -2$, and $n = 10$

1705

34. If the geometric series $54 + 36 + \ldots + \dfrac{128}{27}$ has seven terms in its sum then the value of the sum is

(1) $\dfrac{4118}{27}$

(3) $\dfrac{1370}{9}$

(2) $\dfrac{1274}{3}$

(4) $\dfrac{8241}{54}$

35. A geometric series has a first term of 32 and a final term of $-\dfrac{1}{4}$ and a common ratio of $-\dfrac{1}{2}$. The value of this series is

(1) 19.75

(3) 22.5

(2) 16.25

(4) 21.25

36. Which of the following represents the value of $\displaystyle\sum_{i=0}^{8} 256\left(\dfrac{3}{2}\right)^i$? Think carefully about how many terms this series has in it.

(1) 19,171

(3) 22,341

(2) 12,610

(4) 8,956

9

37. A geometric series whose first term is 3 and whose common ratio is 4 sums to 4095. The number of terms in this sum is

(1) 8   (3) 6

(2) 5   (4) 4

38. Find the sum of the geometric series shown below. Show the work that leads to your answer.

$$27 + 9 + 3 + \ldots \frac{1}{729}$$

$$\frac{29524}{729}$$

39. A college savings account is constructed so that $1000 is placed the account on January 1st of each year with a guaranteed 3% yearly return in interest, applied at the end of each year to the balance in the account. If this is repeatedly done, how much money is in the account after the $1000 is deposited at the beginning of the 19th year? Show the sum that leads to your answer as well as relevant calculations.

$$\$25,116.87$$

40. A ball is dropped from 16 feet above a hard surface. After each time it hits the surface, it rebounds to a height that is $\frac{3}{4}$ of its previous maximum height. What is the total vertical distance, to the nearest foot, the ball has traveled when it strikes the ground for the 10th time? Write out the first five terms of this sum to help visualize.

$$105$$

10